Optimal Design 3D steel trusses by balancing composite motion optimization

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Abstract

The purpose of this senior project is to minimize the weight design of 3D steel trusses by using Balancing Composite Motion Optimization (BCMO), a parameter-free optimization method and effectively simple to use. Without the use of parameters for the optimization process, BCMO eliminates the optimization problems about the selection of unsuitable parameters that are used by the parameter-based algorithm and creates a balance between exploration and exploitation capacities. In addition, BCMO provides a faster convergence rate and much less computing time than the other parameter-based optimization methods. For the steel design, it emphasizes the use of pipe steel design (Round HSS Section) and angle steel design for 3D truss design following the specification for structural steel buildings from [1] an American Institute of Steel Construction (AISC 360-16). The project studies the efficiency and accuracy of BCMO for weight minimization. The project demonstrates the accuracy of the BCMO algorithm by verifying the benchmark problems with the previous studies. Then, it will compare its performance of weight minimization of 3D steel trusses with the PSO algorithm. The results illustrate that BCMO provides better results than the PSO algorithm in computing time, convergence rate, mean weight, and optimum solutions. Finally, the study proves that BCMO can be a good alternative for solving any optimization problems.

Keywords: 3D truss structure, American Institute of Steel Construction (AISC 360-16), Weight minimization, Balancing Composite Motion Optimization (BCMO)

1. Introduction

The meta-heuristic algorithms are widely used due to their utilization of solving complex optimization problems including the size optimization for planar and spatial trusses. [2] The concept of meta-heuristic algorithms is to utilize the search and solving procedures by natural phenomena, human behaviors, and animal behaviors. The purpose of meta-heuristic algorithms is to explore the search space to determine near-optimal solutions with efficiency and obtain the optimum solution from the process. For truss optimization, most of the meta-heuristic algorithms are used to implement the optimization process. [3] Due to many constraints including both equality and inequality constraints and a wide range of design variables, the meta-heuristic algorithms are appropriate alternatives due to the ability to search global minima in high modal and multidimensional spaces.

However, most of the meta-heuristic algorithms are still complicated and time-consuming sometimes. This is due to parameter-dependent problems in the algorithms. These parameters not only obstruct users from effectively tuning the algorithms but also make dealing with various kinds of optimization problems Selecting unsuitable parameters can create an unbalance between exploration and exploitation capacities [4]. [5] For example, if an algorithm focuses on local exploitation, this algorithm may be trapped in local optima. Otherwise, if an algorithm is more focused on global exploration, its convergence speed will be significantly reduced. In addition, it affects the clarity of the method and leads to mistakes during implementation for finding optimum solutions [6].

To find the optimum result without any unbalance conditions, this project illustrates a parameter-free and population-based optimization method called Balancing Composite Motion Optimization (BCMO) inspired by Thang Le Duc et al. [4]. BCMO does not need any parameters for the optimum solution-searching process. In BCMO, the solution space is assumed to be a Cartesian one and the searching movements of candidate solutions are compositely equalized in both global and local ones without any encoding and decoding procedures. The candidate solution can move closer to better ones to exploit the local regions and global regions in the same cartesian space. Therefore, the best-ranked individual in each generation can jump immediately from space to space or intensify its current local space and create a balance between exploration and exploitation capacities. With our interest in the significance of the truss for any building and the process of optimization which leads things to the best, this project, focusing on structural size optimization proposes weight minimization of 3D truss using the new optimization method called Balancing Composite Motion Optimization (BCMO), a Meta-heuristic algorithm. We aim to develop this new methodology to apply it to the truss design of the circular hollow and angle steel section from AISC-LRFD design and obtain efficient design solutions.

2. Literature review

There are many pieces of research about the weight minimization of steel trusses. Tran. [7] proposed the idea of genetic algorithms solving the size optimization problems mainly the planar truss and Dede et al. [8] also proposed weight minimization applied with plane and space trusses by using value and binary encodings with a genetic algorithm (GA). According to the weight minimization and GA, GA requires an enormous size of search space for computing the optimum solution and many parameters to adjust for the weight minimization of trusses. Later, Gomes [9] investigated the use of a particle swarm optimization (PSO) algorithm for the weight minimization of trusses. PSO has proven that it can efficiently search a huge size of search space and discrete design variables with fewer parameters and time-consuming than GA. There are many meta-heuristic algorithms inspired by the nature and social behaviors that also be used for the weight minimization of trusses including the Chaotic coyote algorithm, Turbulent Flow of Water-based Optimization, and Politics Optimizer from Pierezan et al. [10], Khaing [2] and Awad et al. [11], respectively. These algorithms require fewer parameters than PSO algorithms and are proven they can efficiently search a huge size of search space and compute the optimal solutions with discrete and continuous design variables for both complex planar and space trusses. However, all of these algorithms are still parameter-dependent problems in the algorithms. Therefore, there is a research paper about the weight minimization of trusses with parameter-free algorithms called the Jaya algorithm from Degertakin [12]. Nevertheless, Jaya's paper does not include AISC-LRFD for the optimization process. There are many pieces of research that can be related to AISC-LRFD design including the size and shape optimization of trusses from Khaing [2] and Nasrollahi et al [13], respectively. For other structures like steel frames, there are also research papers studying weight minimization by using AISC-LRFD design from Dogan et al. [14] and Ky [15].

For the BCMO algorithm, BCMO has proven that BCMO can provide the optimal solution with efficiency and accuracy based on the previous studies of the BCMO algorithm such as solving the optimization problems of stochastic vibration and buckling behaviors of functionally graded porous microplates with uncertainties of material properties from Tran et al. [16], the optimization design of rectangular concrete-filled steel tube short columns with Balancing Composite Motion Optimization and data-driven mode from Huan et al. [17] and the damage detection in beam structures using Bayesian deep learning and balancing composite motion optimization from Nguyen [18]. For this project, under the specification of AISC-LRFD design, the BCMO algorithm is used to determine the optimum solutions from the weight minimization of steel trusses and eliminate parameter-dependent problems for optimization.

3. Problem Description

In this project, the objective is to minimize the weight of the 3D truss structure. Weight minimization can provide enormous benefits in various aspects including reducing the costs of construction as well as increasing the performance of construction. To formulate the optimization statement, three components, which are the objective function, constraints, and design variables are used to optimize the weight of the truss structure. Cross-sectional areas are considered as discrete design variables and the number of discrete design variables called the search space are from the allowable list from the standard section of Round HSS grade B. The truss design is under the specification of the AISC-LRFD design. The general formulation of the objective function is presented as the single objective function. The objective function is the function to optimize the weight of the structure which is directly relevant to the cross-sectional area. [21] The constraints including inequality constraints are the requirements referred to in the specification of the AISC-LRFD design to examine the safety and serviceability requirements for the optimization process. In addition, the constraints can be applied to the objective function by penalty function to convert the constrained objective function to an unconstrained objective function. In the process of analyzing the optimum solutions, the analysis process applying the BCMO algorithm is calculated by MATLAB program. The result from BCMO algorithms will be compared with the previous studies of the weight minimization of trusses by solving the benchmark problems of the 3D truss structure.

4. Research Procedures and Methodology

In the process of weight minimization of 3D steel trusses, first, it is necessary to research the relevant information sources about the 3D truss and sizing optimization such as the direct stiffness method and the basic concepts of BCMO algorithms. Then, state the optimization problem which is the weight minimization of the truss is the objective function, the design variable is the cross-section area, and the constraints are under the specification of the AISC-LRFD method. Then, apply the BCMO algorithm process with the truss design under the specification of the AISC-LRFD method and examine the constraints violation to obtain the optimum solutions. Then, analyze the results with numerical examples. Then, provide the results of the examples of the discrete optimization benchmark problems of truss structure and illustrate the solution convergence which is from the results and the number of iterations. Then, compare the results of the BCMO algorithms with the various design methods and the previous study of the weight minimization of the truss. Finally, summarize the brief process information of the optimization, efficiency, and accuracy of the result compared with the other methods.

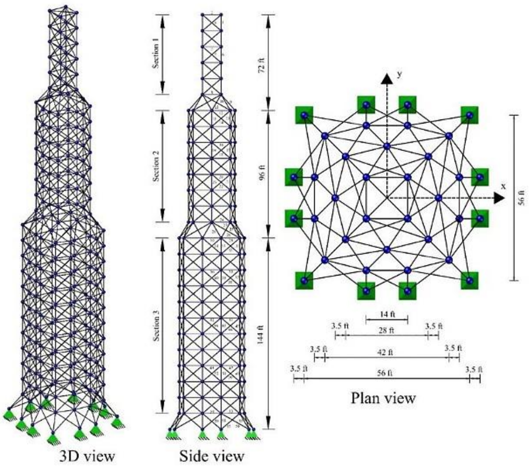

Figure 1. 3D tower truss

5. Optimization statement

The objective function must be subjected to the constraints to formulate the optimization method as follows:

Find A = $\{A_1, A_2, A_3, \ldots\ldots A_{nE}\}$ $A \in D_i$

To Minimize W(A) =

$$\sum_{i=1}^{nE} \boldsymbol{\rho_i A_i L_i} \quad ; i = 1,2,3,\ldots,nE \tag{1}$$

Where A is the vector of the cross-sectional area with nE unknowns, D_i is a set of discrete values with the i^{th} cross-sectional area, ρ is the density of the material, L is the length of the element, A is the cross-section area of the element and nE is the number of the elements.

Subject to:

In the case of tension force, according to AISC-LRFD design, the ultimate tensile force in each element must not exceed the available tensile strength of the element. Therefore, this constraint can be illustrated as follows:

$$\mathbf{P_u \leq \O_t P_n} \tag{2}$$

Where P_u is the ultimate element's tensile force, $\O_t$ is the resistance factor for tension which $\O_t$ = 0.9 and P_n is the nominal tensile strength of the element and $P_n = A_g F_y$ where F_y is the yield stress of steel and A_g is the gross-section area which A_g = A in this study.

In the case of compressive force, according to AISC-LRFD design, the compressive force in each element must not exceed the compressive strength of the element. Therefore, this constraint can be illustrated as follows:

$$\mathbf{P_u \leq \O_c P_n} \tag{3}$$

Where P_u is the ultimate element's compressive force, $\O_c$ is the resistance factor for compression which $\O_c$ = 0.9 and P_n is the nominal compressive strength of the element and $P_n = A_g F_{cr}$ where A_g is the gross-section area which A_g = A in this study and F_{cr} is determined as follows:

$$F_{cr} = \begin{cases} 0.658^{\lambda_c^2} F_y & if\ \lambda_c < 1.5 \\ \frac{0.877}{\lambda_c^2} F_y & if\ \lambda_c \geq 1.5 \end{cases} \quad (4)$$

Where λ_c is defined as follows:

$$\lambda_c = \max \{\frac{KL}{r}\sqrt{\frac{F_y}{E}}\} \quad (5)$$

Where KL, r, and E are the effective buckling length, radius of gyration and modulus of elasticity, respectively. K = 1.0 and radius of gyration are selected from the allowable list from the standard section of Round HSS and angle section.
The limit values of slenderness are illustrated as follows:

$$\lambda_i = \begin{cases} \frac{KL}{r} \leq 300 & for\ tensile\ member \\ \frac{KL}{r} \leq 200 & for\ compression\ member \end{cases} \quad (6)$$

the deflection constraints of each node are given as:

$$\boldsymbol{\delta_i} \leq \boldsymbol{\delta_{max}}\ ;\ i = 1,2,3,\ldots,nN \quad (7)$$

Where $\boldsymbol{\delta_i}$ is the deflection of each node and nN is the number of nodes and $\boldsymbol{\delta_{max}}$ is the maximum deflection of nodes depending on the structure

In this study, the material properties are illustrated as follows:

F_y = 345 MPa, E = 200 GPa, ρ = 7800 kg/m^3

6. Balancing composite motion optimization for weight minimization of the trusses

According to the BCMO's procedure from T. Le Duc [4] and the weight minimization of trusses, we can apply BCMO algorithm step-by-step procedures. First, define the essential components of optimization problems, the number of populations (NP), max generation, the lower bound, upper bound, and the number of design variables of problems. Then, analyze the trusses by direct stiffness method. The AISC-LRFD specifications from equations (2)-(7) are applied as a penalty function. Then, define suitable parameters for the penalize objective function. then, calculate the weight of the structure of individuals in the population. Finally, rank the individuals according to their weight of truss structures.

For BCMO algorithm, first, after ranking the individuals according to their weight of truss structures, the individual that has the least weight of truss in the population $\mathbf{x_1}^{\mathbf{t}}$ is compared to the trial vector, $\mathbf{u_1}^{\mathbf{t}}$. Then, the less weight between the individual and the trial vector is the instant global point $\mathbf{x^t}_{\mathbf{Oin}}$. Then, update the position of the individual from the second individual until all individuals from the number of populations are updated. Then, calculate the weight of the structure of individuals in the population and rank the individuals according to their weight of truss structures. Then, reiterate the trial vector, u_1^t until it reaches the maximum generation. Finally, illustrate the best optimum results.

In **Figure 2**. Illustrates the flowchart of BCMO for weight minimization of the trusses. The flowchart summarized the procedures and algorithm for weight minimization of the trusses as follows:

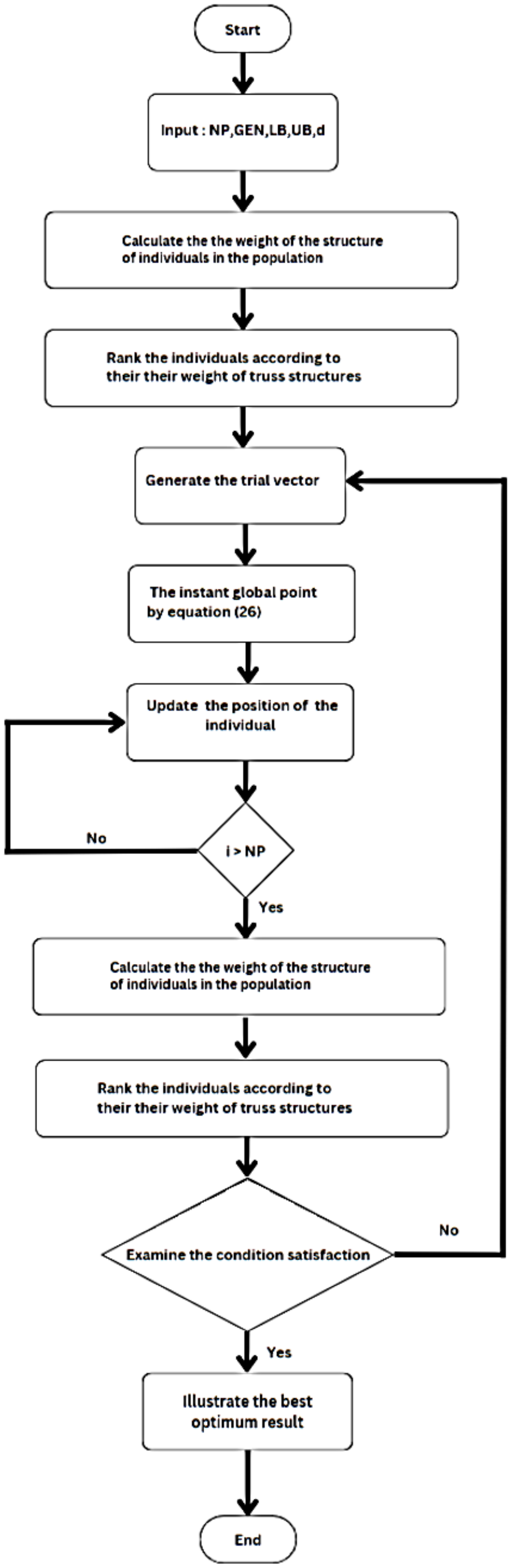

Figure 2. Flowchart of weight minimization of trusses by the BCMO algorithm

7. Numerical results

The objective is to minimize the weight of the truss using discrete variables and design followed by AISC-LRFD. The design variables are cross-sectional areas of all members. For each design example, 10 independent runs were performed by BCMO, and computed the best, average, and standard deviation of the results were presented for each problem. The obtained results of BCMO were compared with the results from the PSO algorithm. The proposed algorithm and direct stiffness method for analysis of truss structures were evaluated in MATLAB and all runs were performed on a 64-bit computer with an Intel Core i5 (2.30 GHz) processor and 8.00 GB of RAM. For this section, 3 examples of the bar space trusses including the spatial 72-bar truss, 397-bar suspension tower truss, and the spatial 942-bar tower truss are examined by the BCMO algorithm, The results are compared with the other's research papers to compare the quality of BCMO's results with the other optimization method as follows:

7.1 The spatial 72-bar truss

This structure is evaluated with the BCMO algorithm to find the optimum result of the truss structure design followed by AISC-LRFD. According to section 2.4 and defines the maximum displacement constraint of 5 mm. The elements and nodes are illustrated in **Figure 3.,** respectively. The loading data case 1 and grouping details are presented in Dede et al. [8] The project divided the problem into 2 case studies: the HSS section and the angle section member.

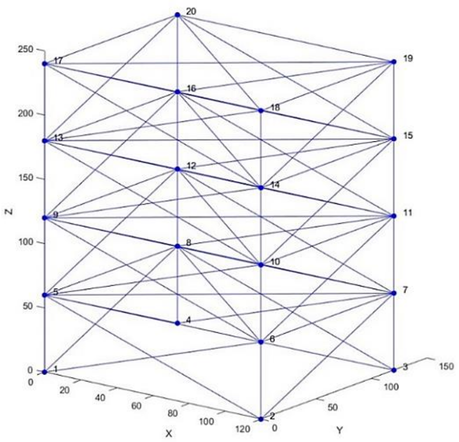

Figure 3. 72- bar space truss's MATLAB Plot

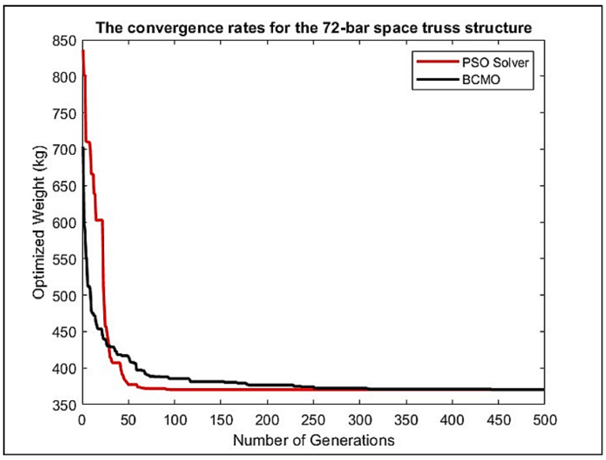

Figure 4. Comparison of the convergence rates of the BCMO algorithms for 72-bar space truss structure (HSS Section)

Table 1. Optimum design comparison for the 72-bar space truss structure (HSS Section)

Best Weight (kg)	370.18	370.18
CV	None	None
Mean Weight (kg)	371.44	371.51
Standard Deviation	0.922	1.616
Number of analyses	1,002,000	1,000,000
Computing Time (seconds)	1,297	1,176

According to **Table 1.** and **Figure 4.**, for HSS section steel members, the best weight is 370.18 kg from both algorithms. Although both algorithms provide the same optimum solution. BCMO computes less time than the PSO algorithm by computing time for 1,176 seconds while PSO computes for 1,297 seconds but PSO provides slightly better in the mean weight and the standard deviation by 371.44 kg and 0.922 respectively.

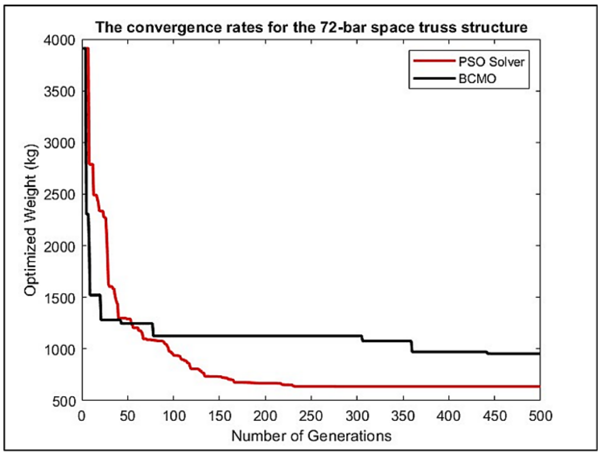

Figure 5. Comparison of the convergence rates of the BCMO algorithms for 72-bar space truss structure (Angle Section)

Table 2. Optimum design comparison for the 72-bar space truss structure (Angle Section)

Best Weight (kg)	634.7484	952.820
CV	None	None
Mean Weight (kg)	742.747	1000.381
Standard Deviation	105.66	66.290
Number of analyses	1,002,000	1,000,000
Computing Time (seconds)	1,278	1,101

According to **Table 2.** and **Figure 5.**, for angle steel members, the best weight is 634.748 kg from the PSO algorithm. PSO significantly decreases the optimum solution till the 250th generation converges to the optimum solution. BCMO's best weight is 952.82 kg which differs by approximately 33.4%. BCMO's convergence rate improves till the 450th generation converges to the optimum solution. Although PSO gives the best results, focusing on the optimization process and data, BCMO has more consistency in optimum solution due to the standard deviation and less computing time than the PSO algorithm by computing time for 1,101 seconds while PSO computes for 1,278 seconds.

To compare the HSS steel section with the angle steel section members, there is an enormous difference in the best weight in the PSO algorithm and a fair difference in the BCMO algorithm. Due to different sets of discrete design variable between HSS steel section and angle steel section members, the optimum solutions from both algorithms are also different. Nevertheless, according to the results, PSO still provides better optimum solutions and the mean weight than BCMO and converges to the optimum solution faster than BCMO.

7.2. The 397-member, 220 kV suspension tower

The 397-suspension tower truss is used to verify these algorithms to examine the truss that is bigger than the 72-bar spatial truss and can be utilized. The 397-suspension tower truss is made up of 397 members with 31.5 m. high. It is a lattice tower with two earth-wire peaks to carry the optical communication wires and the support structure for overhead transmission lines to support the conductors and lightning conductors. The structure consists of 397 members of steel sections and 120 nodes. The elements and nodes are illustrated in Figure 4.13. and Figure 4.14., respectively. The loading data are referred from Tort [22].

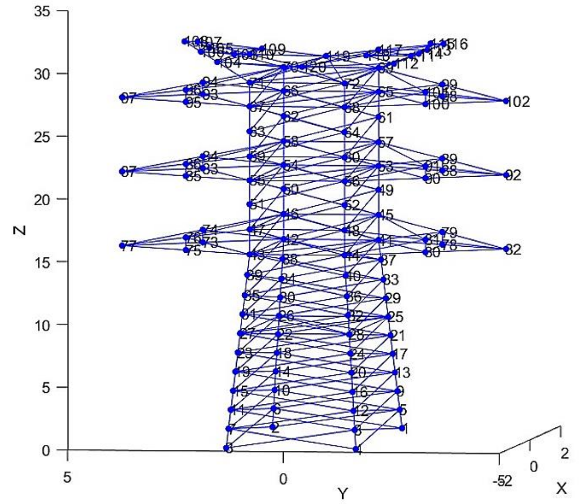

Figure 6. 397-bar suspension tower truss's MATLAB Plot

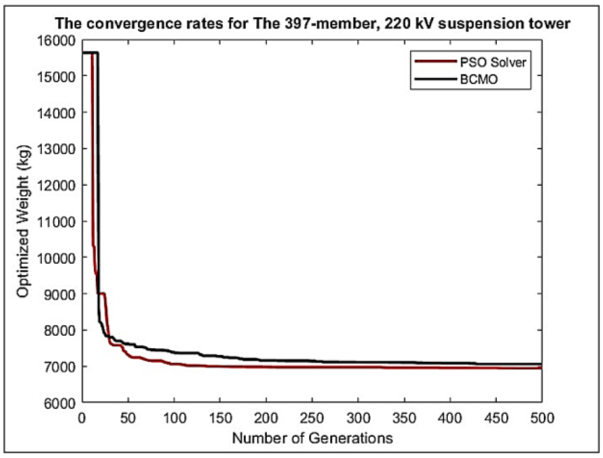

Figure 7. Comparison of the convergence rates of the BCMO algorithms for 397-bar suspension tower truss structure (HSS section members)

Table 3. Optimal design comparison for the 397-suspension tower truss structure (HSS section members)

Best Weight (kg)	6,949.6	7,059.2
Mean Weight (kg)	7,105.400	7,430.141
Standard Deviation	76.594	51.580
Number of analyses	1,002,000	1,000,000
Computing Time (seconds)	6,457	6,178

According to **Table 3.** and **Figure 7.**, for HSS Section, the best weight is 6,949.6 kg from PSO algorithm while BCMO's best weight is 7,059.2 kg which differs approximately 1.55 % from BCMO algorithms. BCMO's and PSO's convergence rate remain stable from the start during initialized phase then. Both algorithms improve significantly till the 250th generation converges to optimum solutions. Although PSO gives the best results, BCMO has less computing time than PSO algorithm by computing time for 6,178 seconds while PSO computes for 6,457 seconds. PSO provides better the optimum solution and the mean weight for this problem.

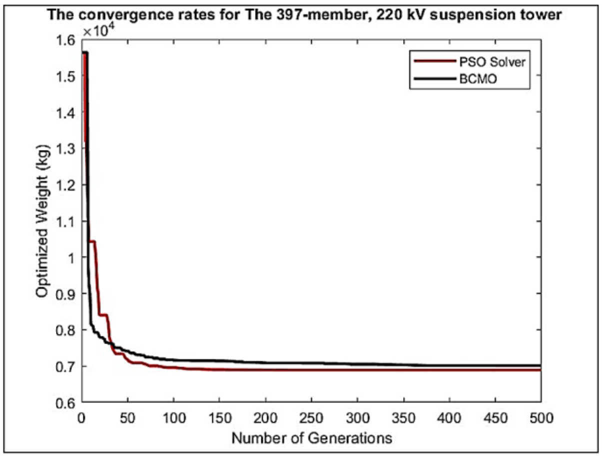

Figure 8. Comparison of the convergence rates of the BCMO algorithms for 397-bar suspension tower truss structure (Angle section members)

Table 4. Optimum design comparison for the 397-suspension tower truss structure (Angle section members)

Best Weight (kg)	6,893.8	7,009.9
Number of analyses	2,505,000	2,500,000
Computing Time (seconds)	17,662	14,465

According to **Table 4.** and **Figure 8.**, for angle section members, the best weight is 6893.8 kg from PSO algorithm while BCMO's best weight is 7009.9 kg which differs approximately 1.66% from BCMO algorithms. PSO's convergence rate remains the same during the initialized phase and converges to the optimum solution in the 150th generation, while BCMO's convergence rate remains the same during the initialized phase and converges to the optimum solution in the 250th generation Although PSO gives the best results, focusing on the optimization and data, BCMO has less computing time than PSO algorithm by computing time for 14,465 seconds while PSO computes for 17,662 seconds.

To compare both cross sections area, The best weights are similar are 6949.6 and 6893.8 kg. The PSO algorithm gives slightly better results for both sections while the BCMO algorithm gives better time in both sections and there is an enormous difference between both steel sections.

7.3 The spatial 942-bar tower

The large number of truss members are considered to evaluate BCMO's efficiency and accuracy. The structure consists of 942 members of steel sections, and 244 nodes. The elements and nodes are illustrated in **Figure 9**. For this structure, the cross-section areas of the 942-bar's members of the truss are classified into 59 groups as the number from Figure 4.18. The optimization process is under the optimization statement according to section 2.4 and defines the maximum displacement constraint of 15 in. referred from Degertekin [23].

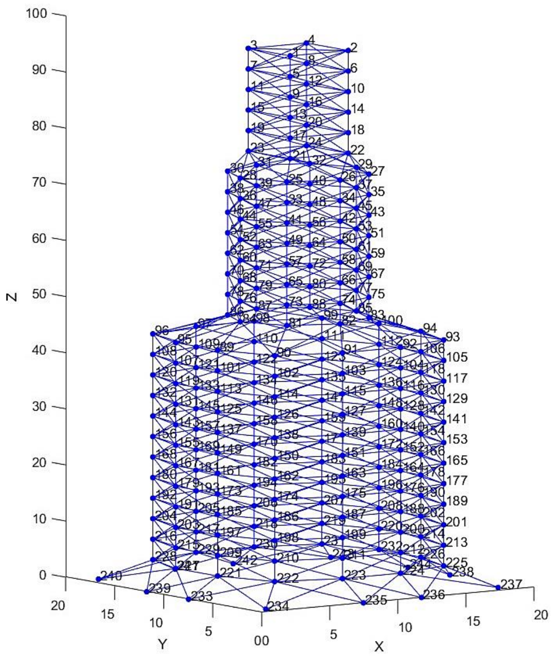

Figure 9. 942-bar spatial tower truss's MATLAB Plot

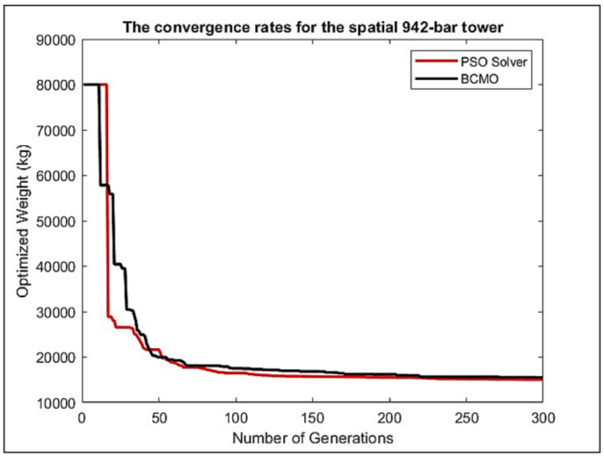

Figure 10. Comparison of the convergence rates of the BCMO algorithms for 942-bar spatial tower truss structure (HSS section members)

Table 5. Optimal design comparison for the 942-spatial tower truss structure (HSS section members)

Best Weight (kg)	15680	16019.453
Mean Weight (kg)	16882.6882	16450.137
Standard Deviation	827.3676251	536.8350686
Number of analyses	60,200	60,000
Computing Time (seconds)	1516.722	1297.112

According to **Table 5**. and **Figure 10.**, for HSS members, the best weight is 15680.00 kg from PSO algorithm while BCMO's best weight is 16019.453 kg which differs approximately 2.16 % from PSO algorithms. BCMO's and PSO's convergence rate remain stable from the first generation until 20th generation and start to significantly improve the optimum solution till the 200th generation to converge to optimum solutions. Although PSO gives the best results, focusing on the optimization process and data, BCMO has more consistency in optimum solution due to the standard deviation and less computing time than PSO algorithm by computing time for 1297.112 seconds while PSO computes for 1516.722 seconds.

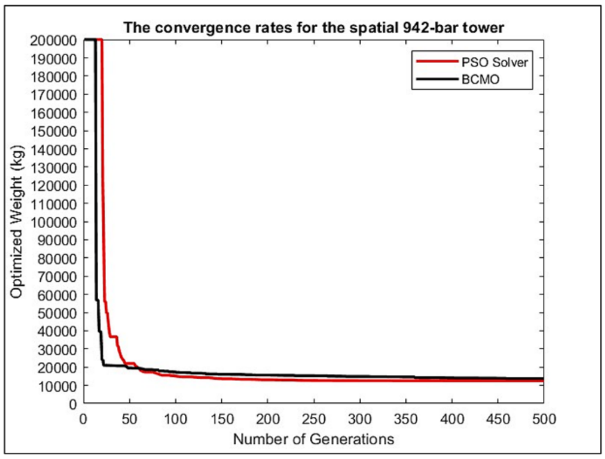

Figure 11. Comparison of the convergence rates of the BCMO algorithms for 942-bar spatial tower truss structure (angle section members)

Table 6. Optimal design comparison for the 942-spatial tower truss structure (angle section members)

Best Weight (kg)	12,494	13,740.17
Number of analyses	1,002,000	1,000,000
Computing Time (seconds)	25754	21517

According to **Table 6**. and **Figure 11**., for HSS members, the best weight is 12,494 kg from PSO algorithm while BCMO's best weight is 13,740.17 kg which differs approximately 9.07 % from PSO algorithms. BCMO's and PSO's convergence rate remain stable from the first generation until 50th generation and start to significantly improve the optimum solution till the 350th generation to converge to optimum solutions. BCMO has less computing time than PSO algorithm by computing time for 21517 seconds while PSO computes for 25754 seconds.

8. Conclusion

The project presents the weight minimization of 3D steel truss under applied force and constraints of AISC-LRFD design by using the new meta-heuristic optimization found in 2020 called "Balancing Composite Motion Optimization" or BCMO algorithm. The project aims to study the performance in terms of efficiency and accuracy of the algorithm by comparing BCMO with PSO algorithm. The project evaluates BCMO algorithm's accuracy with 3 benchmarks problem such as the 5-bar statically determinate truss, the 6-bar statically determinate truss and the 25-bar space truss and compares with PSO and the previous study from the others. In this part, we set the default setting of GA and PSO. The result is BCMO can give the exact solution like GA and PSO. In addition, BCMO can give less computing time and a smaller number of analyzes than these methods.

For test problems in section 4.2, the project uses 3D truss such as the 72-bar space truss, 397-bar suspension tower truss, and 942-bar spatial tower truss as the benchmark problems. We evaluate the truss with HSS and angle steel section members for these problems except 942-bar spatial tower truss. Setting the parameters for both PSO and BCMO algorithm, if we evaluate the enormous truss members, a lot of number of population sizes and the small amount of constrain violation parameter are required for the problems. The results for these test problems are for all the problems. According to the results, the project found that the more the number of population size, the more probability of getting fewer optimum results and more computing time for BCMO's optimization process. PSO give the better results in optimum weight, the mean weight and the convergence rate. This is due to the efficiency of balancing between exploration and exploitation. However, from the results, BCMO can give the optimum solution faster than PSO. It takes less computing time, and the optimum solutions in each generation are more consistent than PSO which mean there are more adjacent optimum solution in each generation than PSO.

According to the results from test problems, BCMO may be unsuitable for the large-scale of problem comparing with PSO algorithm, however, with the parameter-free that easily usable and less-computing time, if BCMO is develop in the future, the algorithm could be one of the most usable optimizations in the future.

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